Math 252 Practice Final

1. Find parametric equations of the tangent line to the curve $c(t) = \left\langle e^t \cos t, e^t \sin t, e^t \right\rangle$ at $t = 0$.

2. Find an equation of the plane that contains the point (2, 0, 3) and the line $x = -1 + t, \quad y = t, \quad z = -4 + 2t$.

3. Find the first order partial derivatives of $f \circ g$ if $f(u,v,w) = \left\langle u^2 - v^2, 3w, uvw \right\rangle$ and $g(x,y) = \left\langle x, -y, xy^2 \right\rangle$.

4. Find the tangent plane to the graph of $f(x,y) = x^{\frac{1}{2}} + y^{\frac{1}{2}}$ at the point (4, 9, 5).

5. Find and classify all the relative extrema of $f(x,y) = xe^y$.

6. Find the maximum and minimum values of $f(x,y) = x^2 - y$ subject to the constraint $x^2 + y^2 = 25$.

7. Find the arclength of $c(t) = \left\langle \sin\left(e^t\right), \cos\left(e^t\right), \sqrt{3}e^t \right\rangle$ for $0 \le t \le 1$.

8. Find the unit tangent vector to the curve $c(t) = \left\langle e^t, e^{-t}, t \right\rangle$ at t = 0.

9. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} \cos\left(x^2 + y^2\right) dx dy$.

10. Use a triple integral to find the volume of the solid bounded by the sphere $x^2 + y^2 + z^2 = 8$ and the planes z = 0 and z = 2.

11. Find the surface area of the portion of the surface $z = xy$ that is above the sector in the first quadrant bounded by the lines $y = \frac{x}{\sqrt{3}}$, y = 0, and the circle $x^2 + y^2 = 9$.

12. Evaluate $\int_C -ydx + xdy$ along $y^2 = 3x$ from the point (3, 3) to the point (0, 0).

13. Use Green's theorem to evaluate $\int_C x^2 ydx - y^2 xdy$ where C is the boundary of the region in the first quadrant enclosed between the coordinate axes and the circle $x^2 + y^2 = 16$.

14. Use Gauss' Divergence theorem to evaluate $\iint_S F \cdot dS$ if $F(x, y, z) = (x, y, z)$ and S is the surface of the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the xy-plane.